# Control Lec 4

Bode diagram

Stability

① Absloute stability

- Routh

Relative stability

* Bode diagram
* Polar plot
* Nyquist

→ Relative stability: To what range or to what degree the system is stable through stability indicators [phase margin (PM) and gain margin (GM)]

## Frequency analysis

→ given T.F in s-domain & we want to describe it in freq. domain

we replace $s \longrightarrow j\omega$

Given o.L.T.F $GH(s)$ to get Bode diagram

① Replace $s \longrightarrow J\omega$

$$GH(J\omega) = GH(s)\Big|_{s \to J\omega}$$

② $\underbrace{|GH(J\omega)|}_{\text{magnitude}}$ , $\underbrace{\angle GH(J\omega)}_{\text{Angle}}$

③ $|GH(J\omega)|_{dB} = 20 \log |GH(J\omega)|$

dB → ديسيبل

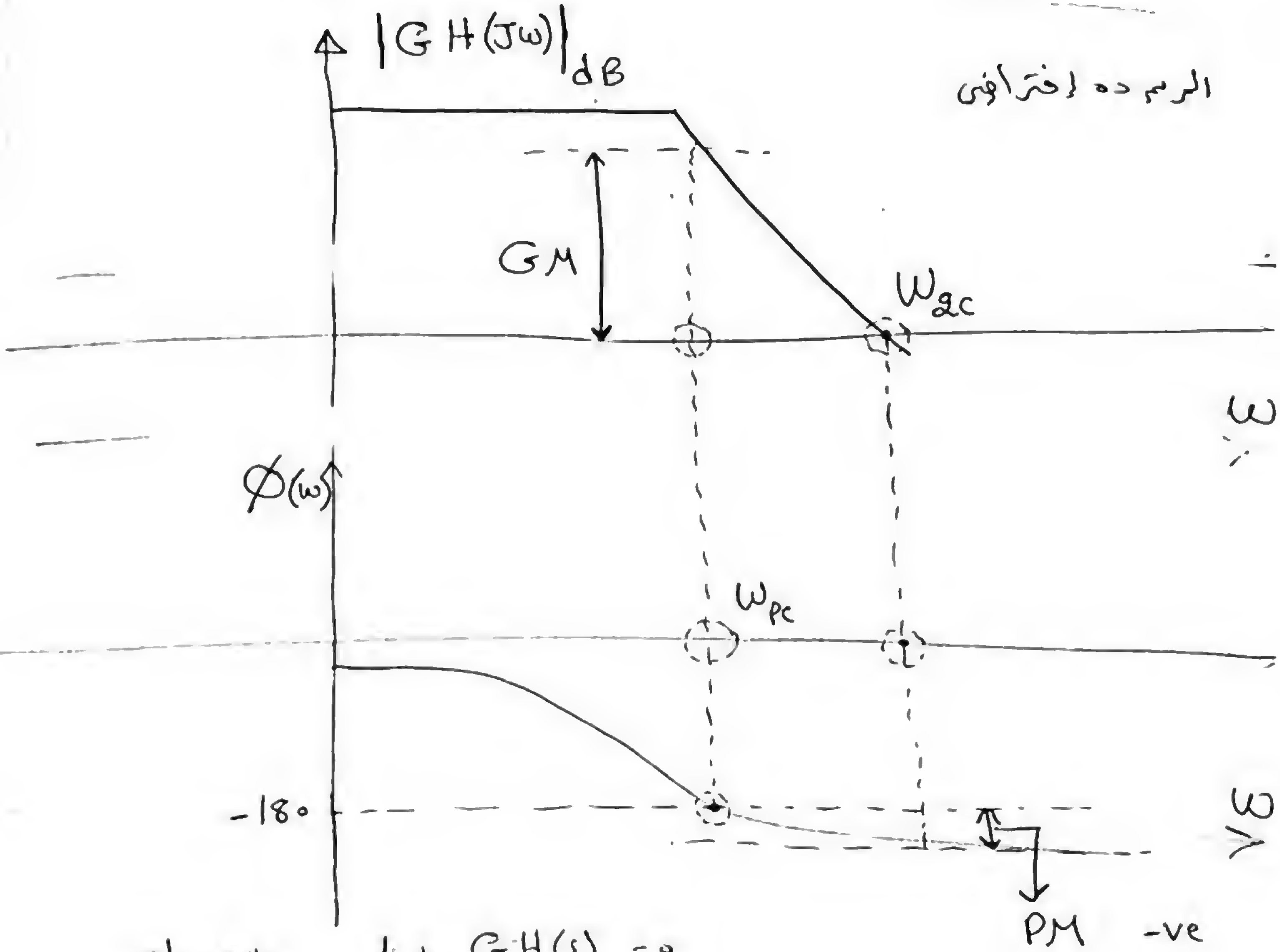
$\boxed{2}$ Lec 1

لأن الرسمة هتكون كبيرة فلازم تصغرها.

$$|\quad| = \sqrt{(Real)^2 + (img.)^2}$$

$$\phi = \tan^{-1}\left(\frac{imag.}{Real}\right)$$

الرسم ده افتراضي




**ch. eq** $\quad 1 + GH(s) = 0$

$$GH(s) = -1 + J0$$

$$\therefore \left.\left|\;\;\right|\right._{dB} = 0$$

$$\angle GH(J\omega) = -180$$

جاية من ال ch. eqn

هنحسب ال (stability) منها .

$W_{pc}$ ≡ phase cross freq.

$GM = -ve \longrightarrow$ عشان فوق ال Zero dB

$W_{gc}$ ≡ gain cross freq. ← التردد المناظر لـ gain مقداره Zero dB

$PM = -ve$ → phase margin

→ لأنها تحت الزاوية $180°$

## stability

① GM & PM

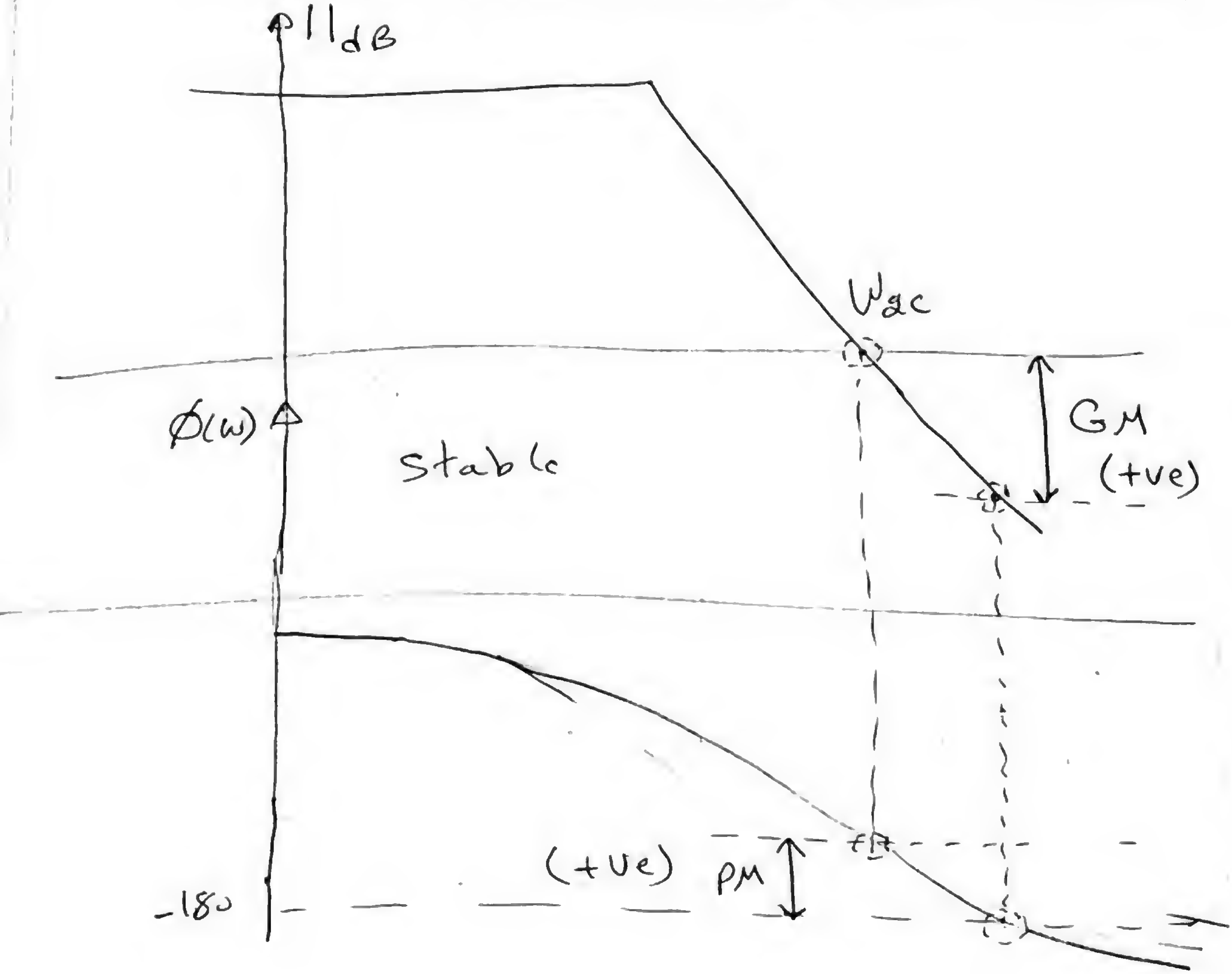
a) $> 0$ (+ve) → stable

b) $< 0$ (−ve) → unstable

c) $= 0$ critical stable

**Note**

$$|GH(j\omega)|_{dB} = 20 \log |GH(j\omega)|$$




$$\boxed{PM = 180 + \phi(\omega)\Big|_{\omega=\omega_{gc}}}$$

* The open loop T.F $GH(s)$ Can be in the following forms:-

① $GH(s) = K$

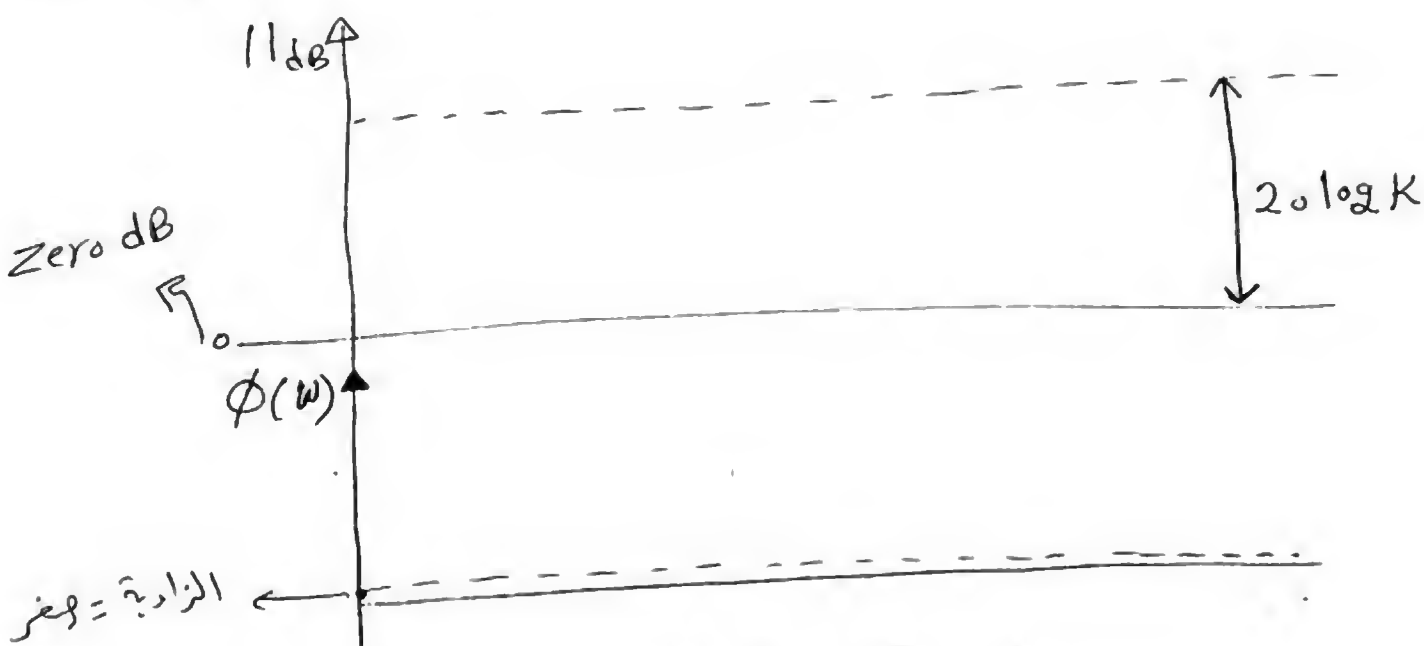
a) $s \rightarrow J\omega$ $\qquad GH(s) = GH(J\omega) = K$

ثابت ملوش دعوة بأي تغيير

b) $|GH(J\omega)| = K$

c) $|GH(J\omega)|_{dB} = 20 \log K$

d) $\phi(\omega) = \tan^{-1}\left(\frac{0}{K}\right) = 0$



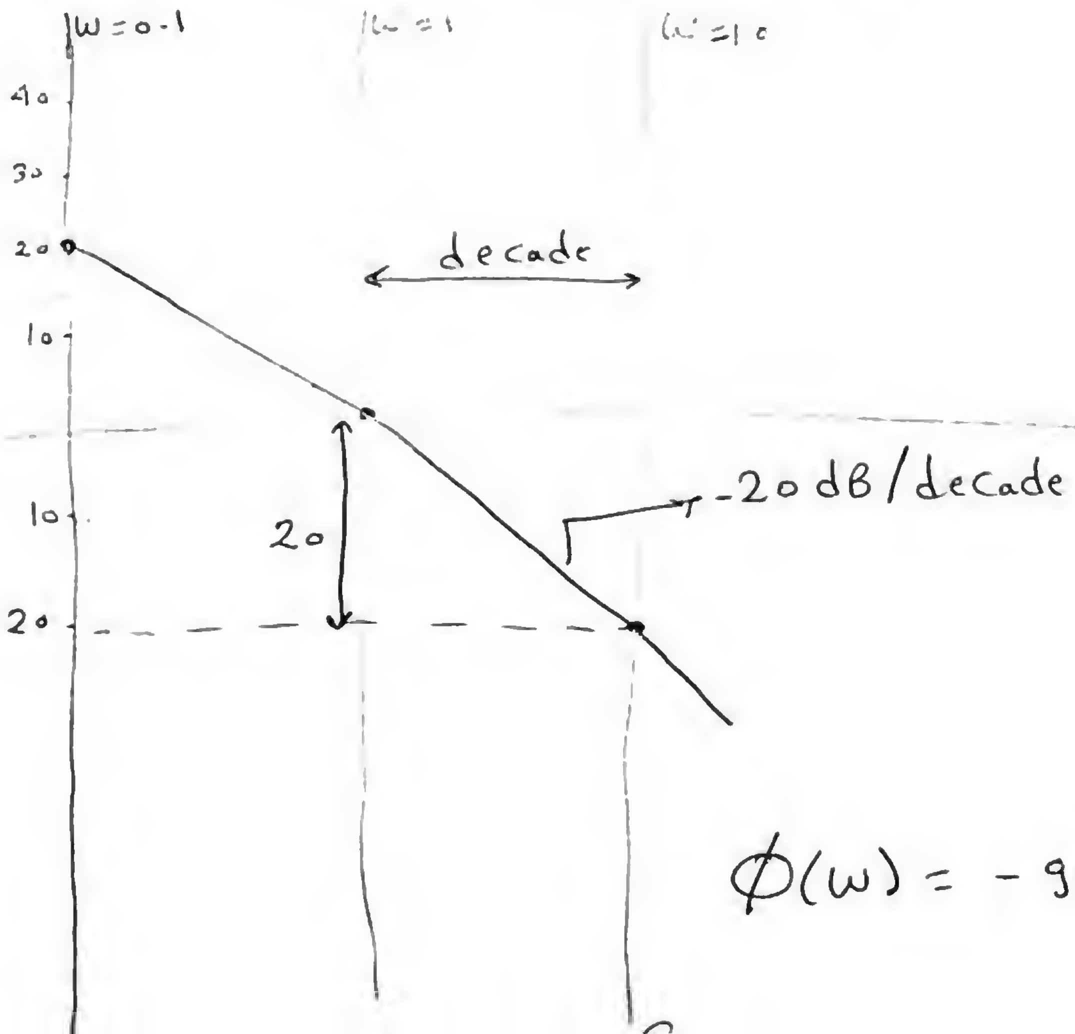
② $GH(s) = \frac{1}{s}$

a) $s \to J\omega \Rightarrow GH(J\omega) = \frac{1}{J\omega}$

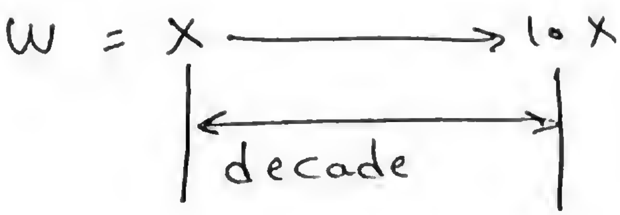
b) $|GH(J\omega)| = \frac{1}{\omega}$ (c) $|GH(J\omega)|_{dB} = 20\log\left(\frac{1}{\omega}\right)$

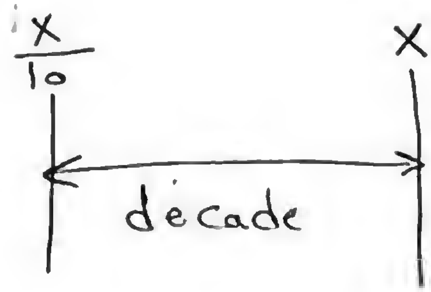
$\Rightarrow$ $|GH(J\omega)|_{dB} = 20\log\omega^{-1} = -20\log\omega$

| $\omega$ | 0.1 | 1 | 10 | 100 |
|---|---|---|---|---|
| $\lvert\,\rvert_{dB}$ | 20 | 0 | -20 | -40 |



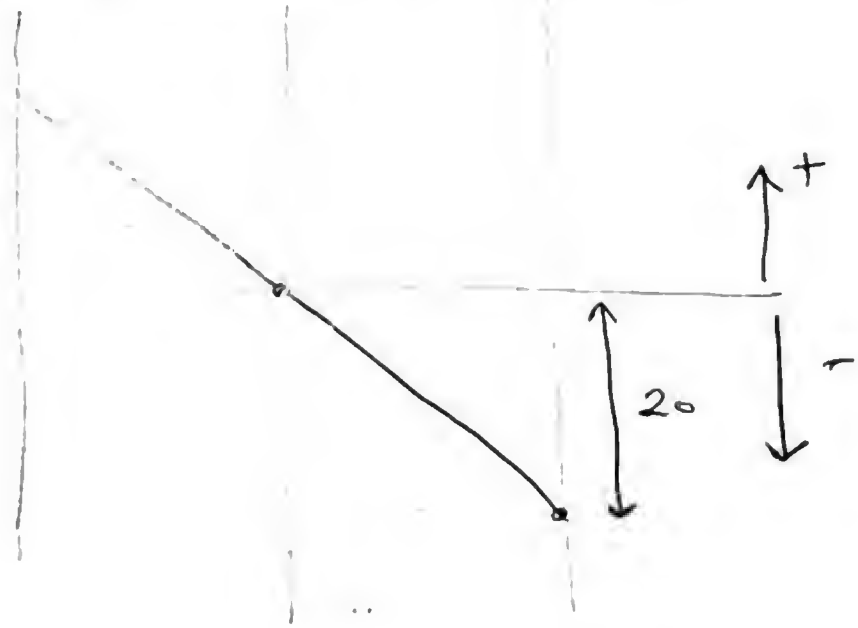

$\phi(\omega) = -90^\circ$



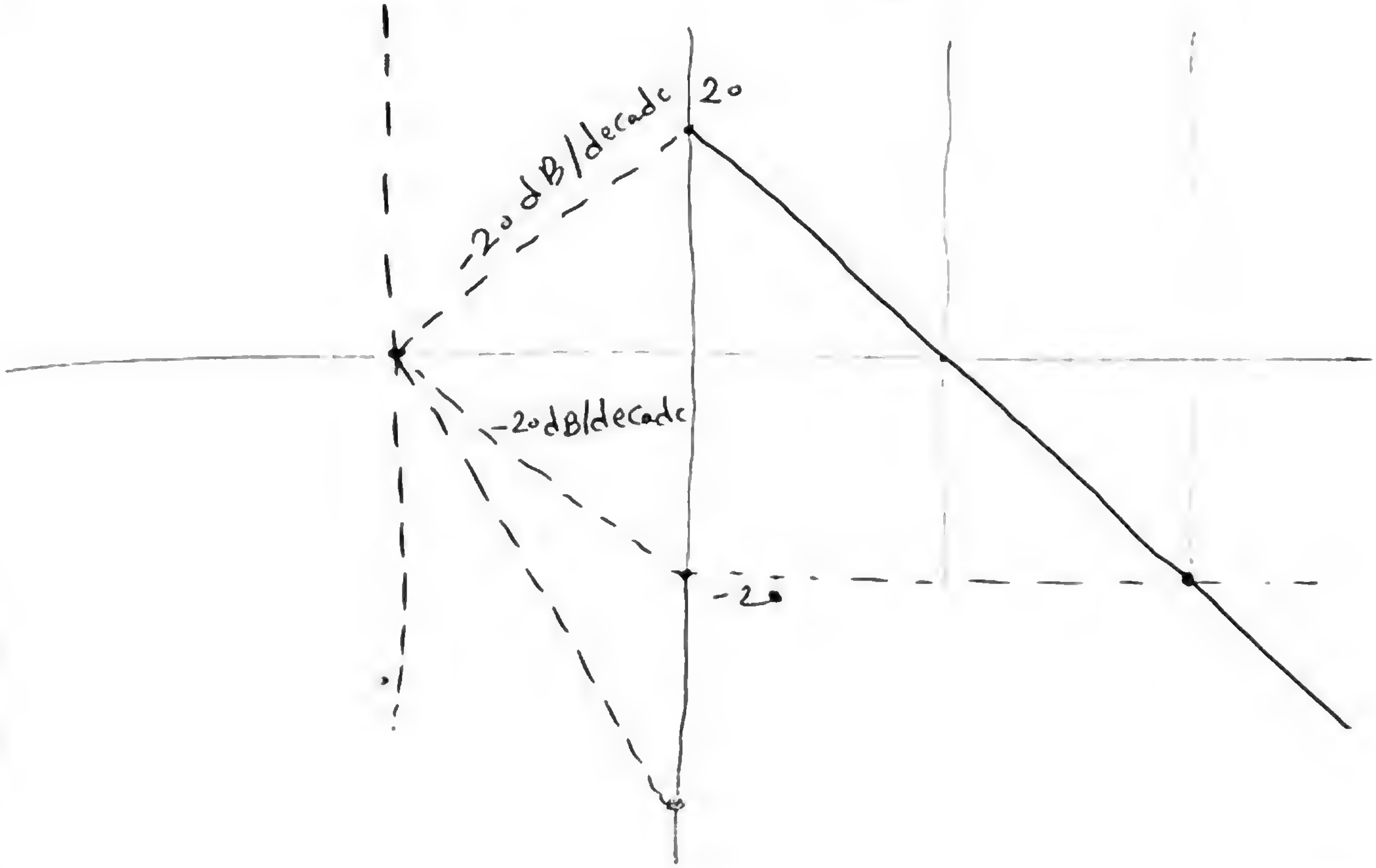

← في ورق ال (Semi log) فهو بيتقسم مربعات كبيرة
كل مربع عبارة عن (decade) وتبدأ من عند (w=value)
مش هتبدأ عند w=0.

← لو الحالة دي جاتلك عندك خط ميله -20 dB/decade
⇐ تعرف انه خط مستقيم لو عرفت عليه نقطة تجيب الباقي.




← خد (decade) يمين وآخر شمال
← الميل بالسالب انزل لتحت بالقيمة 20 ولو الموجب تطلع لفوق.

ـــ أو حل آخر (إنك تعمل (decade) هامش.



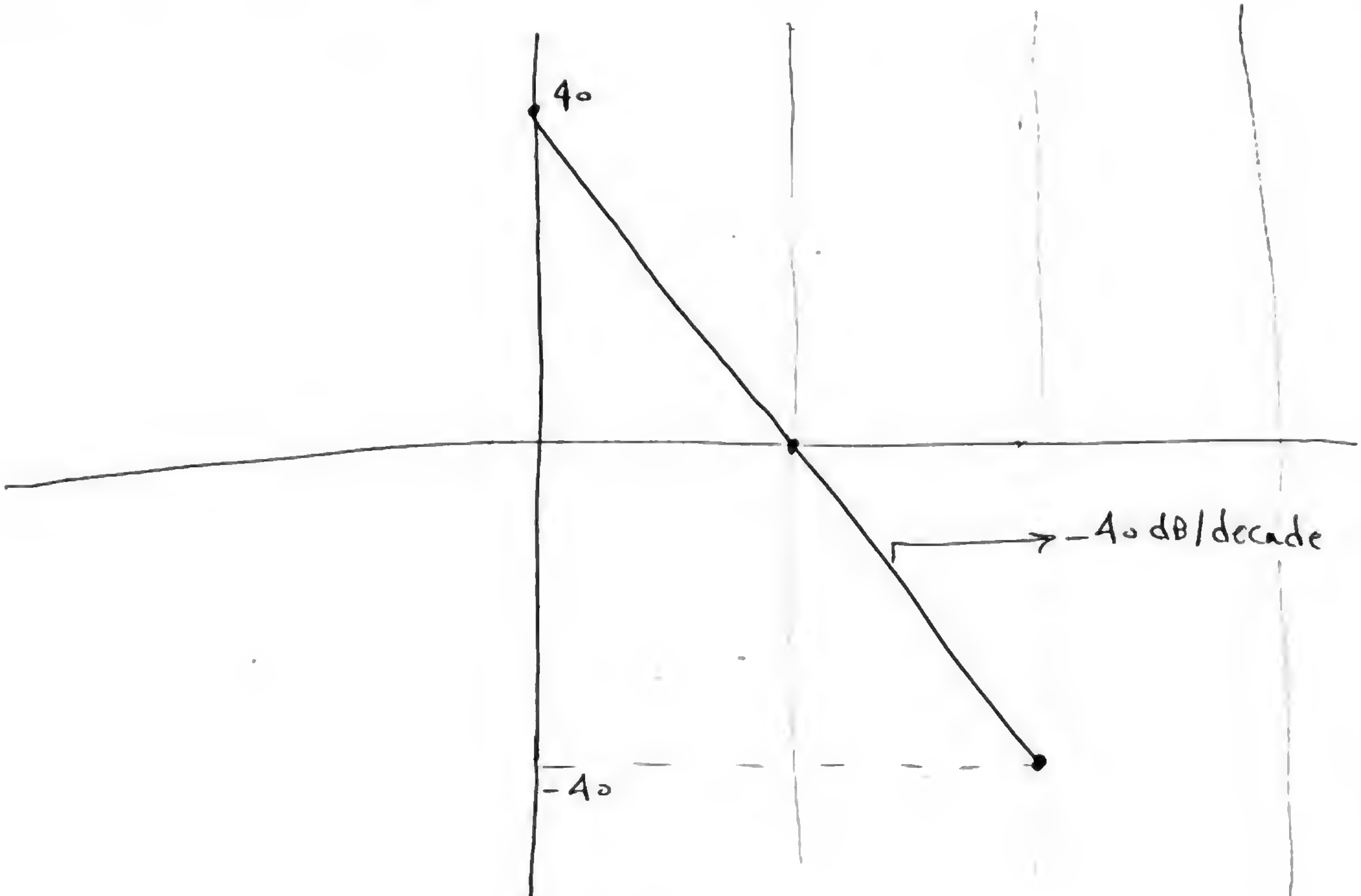

③ $GH(s) = \dfrac{1}{s^2}$

a) $s \longrightarrow j\omega$ $\qquad GH(j\omega) = \dfrac{1}{(j\omega)^2}$

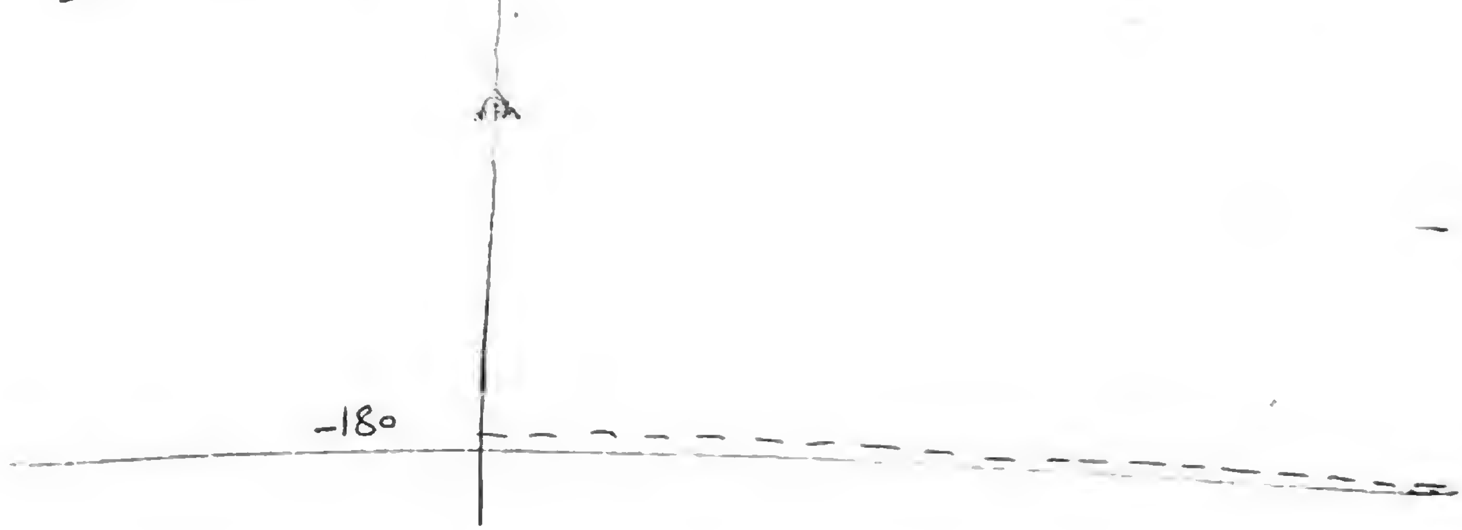
b) $\left| GH(j\omega) \right| = \dfrac{1}{\omega^2}$

c) $\left| \quad \right|_{dB} = 20 \log \dfrac{1}{\omega^2} = 20 \log \omega^{-2}$

$= -40 \log \omega$




$$\frac{1}{s^2} \Rightarrow \phi(\omega) = -180$$



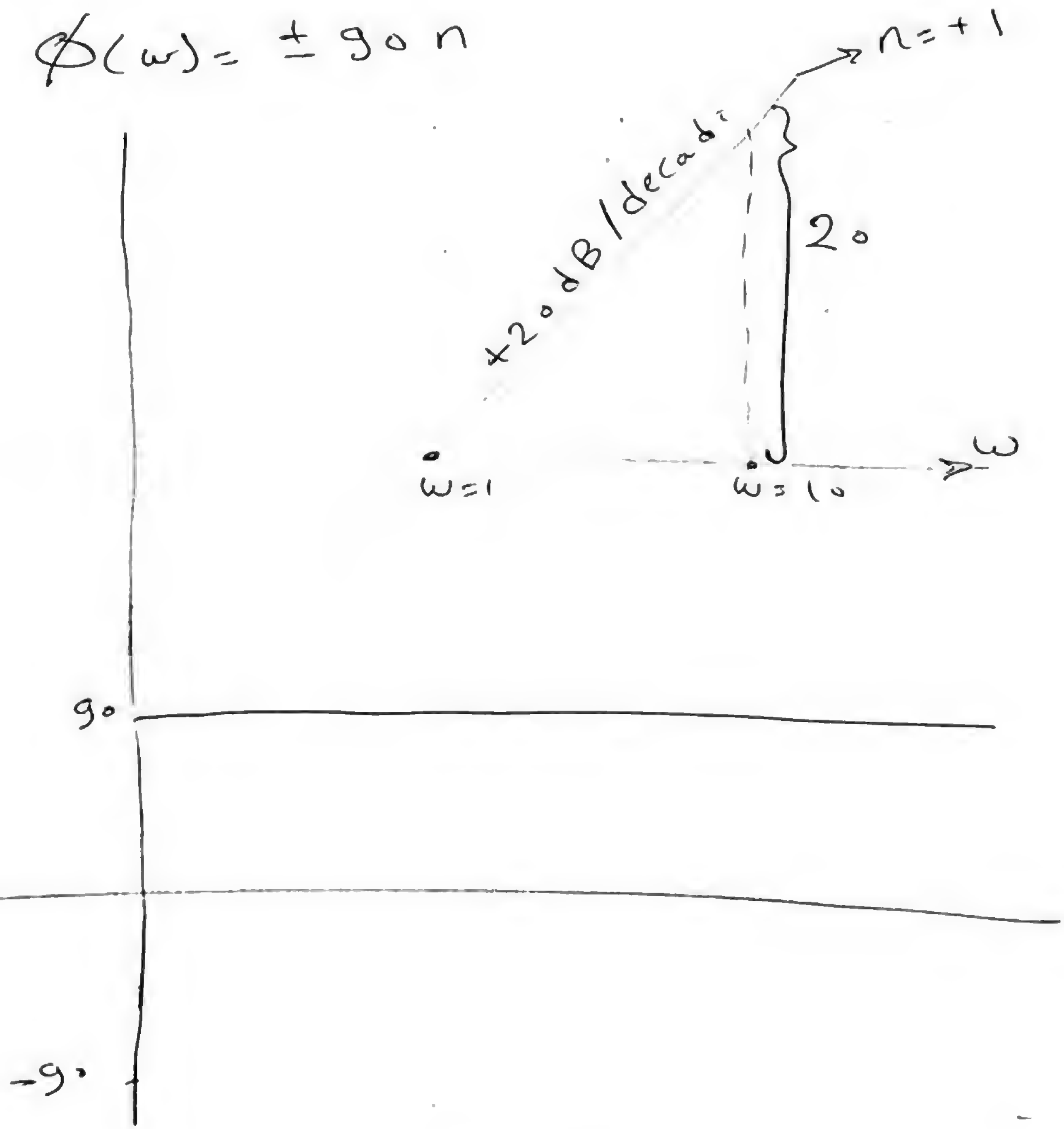
$$GH(s) = (s)^{\pm n}$$

$| \, |_{dB} \Rightarrow$ خط مستقيم يمر بـ $\omega = 1$ ميله

البسط $\rightarrow \pm 20n \; dB/decade$

المقام

$$\phi(\omega) = \pm 90n$$



$$\boxed{9} \quad GH(s) = \frac{K}{s}$$

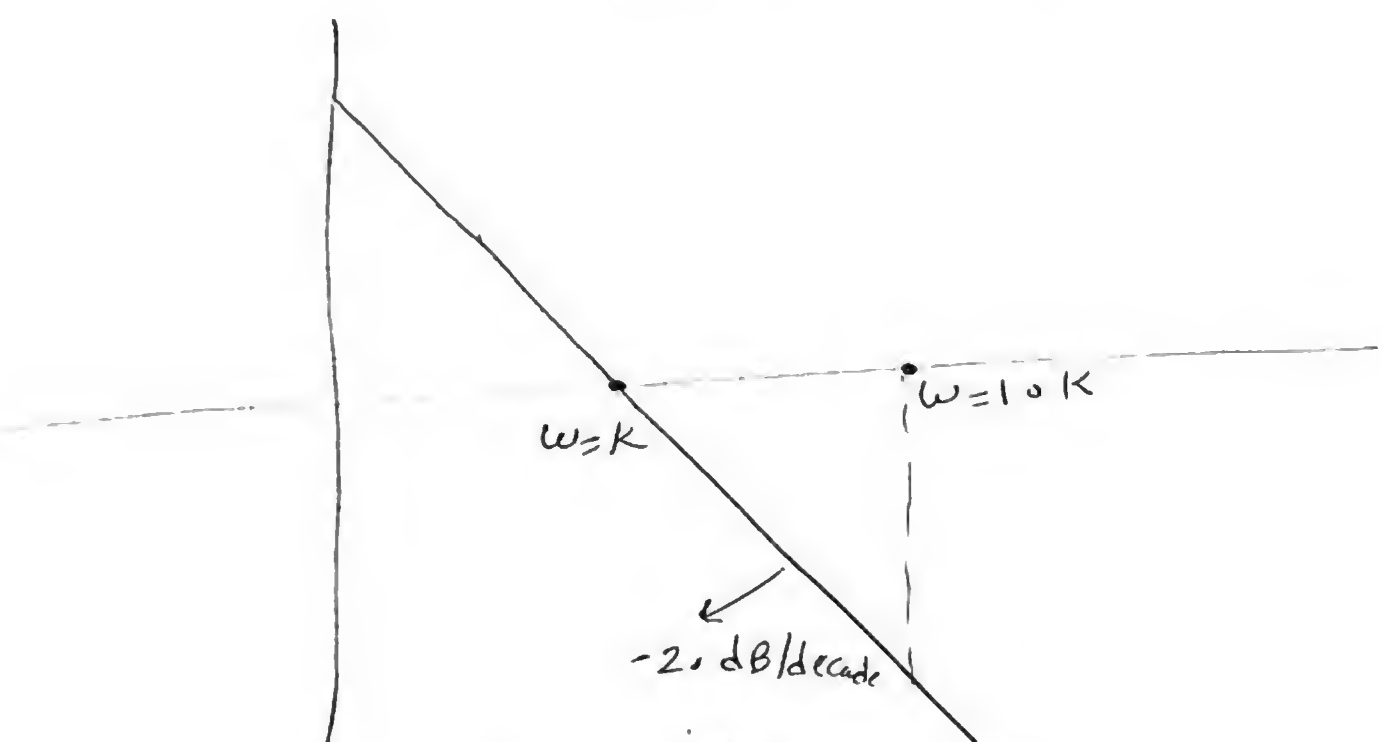
$$s \longrightarrow j\omega \qquad GH(j\omega) = \frac{K}{j\omega}$$

$$|GH(j\omega)| = \frac{K}{\omega}$$

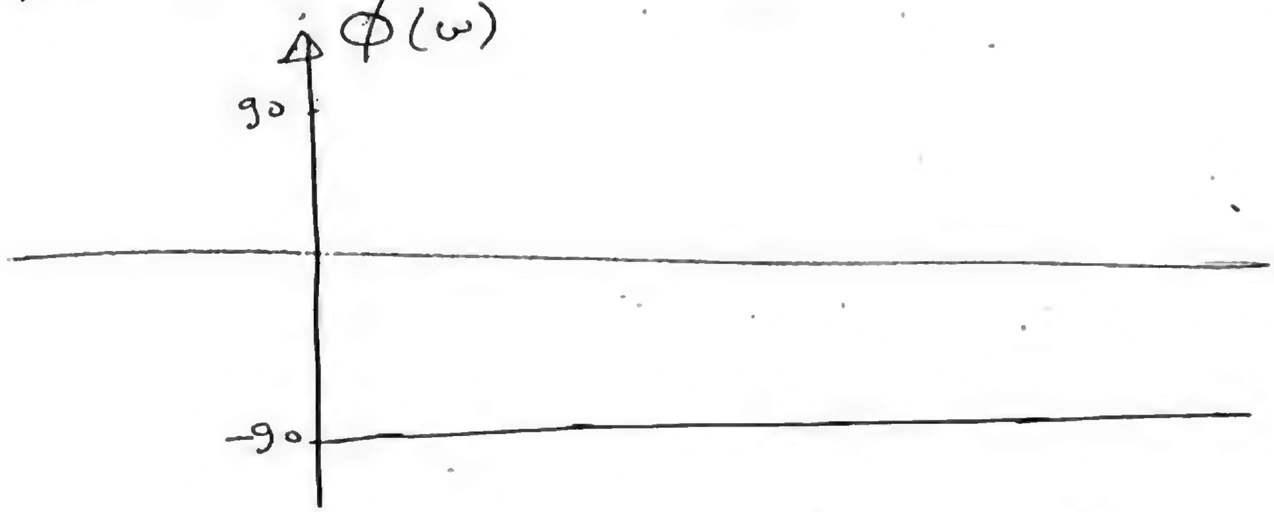
$$|GH(j\omega)|_{dB} = 20 \log\left(\frac{K}{\omega}\right)$$

$$= 20 \log\left(\frac{\omega}{K}\right)^{-1} = -20 \log\left(\frac{\omega}{K}\right)$$

← نقطة مرور الخط هو ما زال خط مستقيم بنفس الميل
لكنه يمر بالنقطة $W = K$



$\phi(\omega) = -90$




$$* \quad GH(s) = \frac{K}{s^2}$$

ـــ خط مستقيم يمر $\omega = \sqrt{K}$ ميله $= -40$ dB/decade

$$\phi(\omega) = -180^\circ$$

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5) $$* \quad GH(s) = \frac{K}{s^3}$$

ـــ خط مستقيم يمر بالنقطة $\omega = \sqrt[3]{K}$ ميله

$-60$ dB/decade ،

$\phi(\omega) = -270^\circ$

[6]

$$GH(s) = \left(1 + \frac{s}{c}\right) \quad , c \rightarrow \text{Constant}$$

a) $s \longrightarrow J\omega$

$$GH(J\omega) = \left(1 + J\frac{\omega}{c}\right)$$

$$|GH(J\omega)| = \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$$

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{\left(1 + \frac{\omega}{c}\right)^2}$$

approximation

① $\omega < c \implies \left(\frac{\omega}{c}\right)^2 \ll 1$ (neglect)

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1+0} = 0$$

② $\omega > c \implies \left(\frac{\omega}{c}\right)^2 \gg 1$ ∴ ($1 \rightarrow$ neglect)

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$= 20 \log \frac{\omega}{c}$$

[6]

$$GH(s) = \left(1 + \frac{s}{c}\right) \quad , c \rightarrow \text{constant}$$

a) $s \longrightarrow J\omega$

$$GH(J\omega) = \left(1 + J\frac{\omega}{c}\right)$$

$$|GH(J\omega)| = \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{c}\right)$$

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{\left(1 + \frac{\omega}{c}\right)^2}$$

approximation

① $\omega < c \Rightarrow \left(\frac{\omega}{c}\right)^2 \ll 1$ (neglect)

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1+0} = 0$$

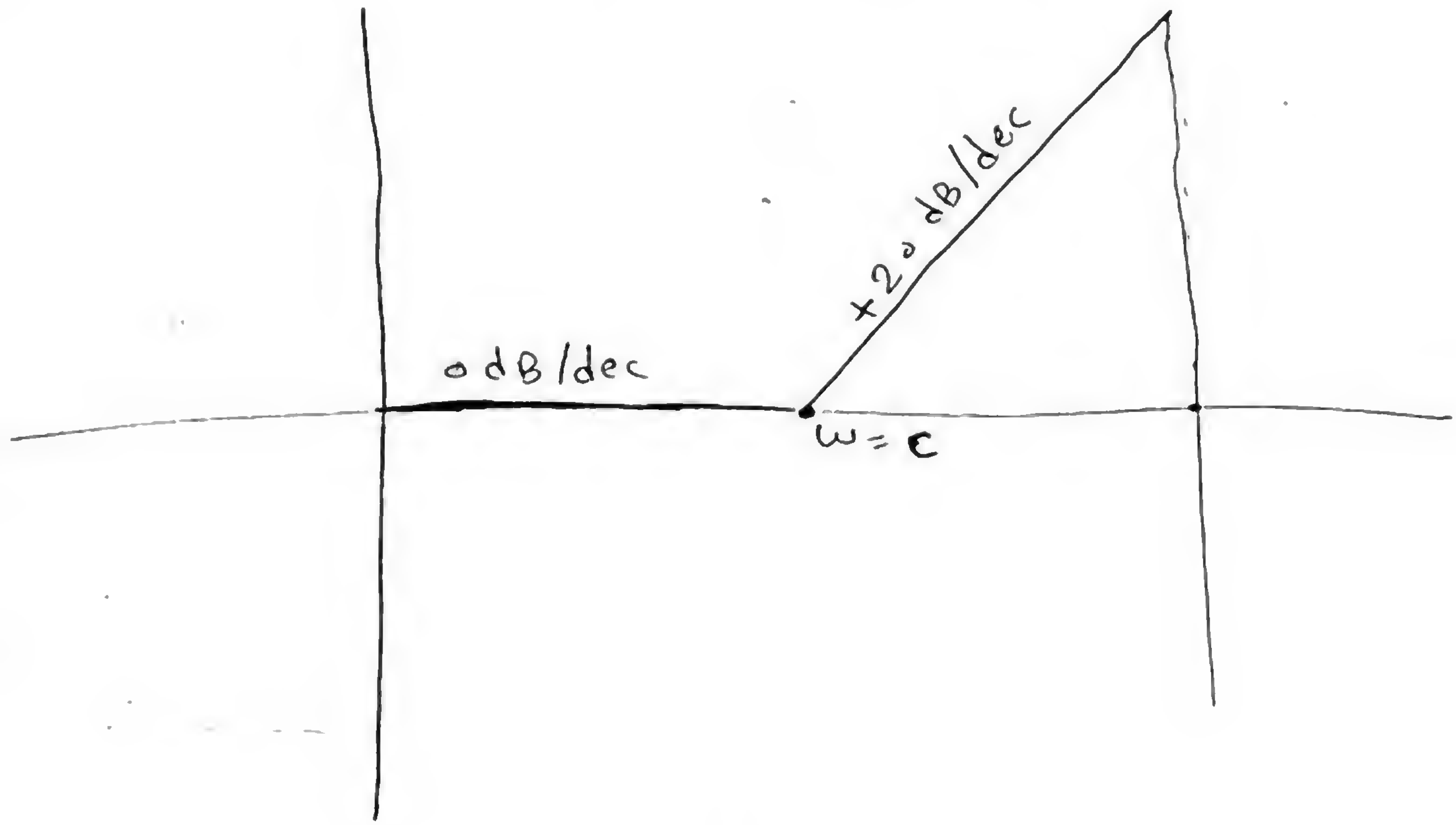
② $\omega > c \Rightarrow \left(\frac{\omega}{c}\right)^2 \gg 1$ $\therefore$ ($1 \rightarrow$ neglect)

$$|GH(J\omega)|_{dB} = 20 \log \sqrt{1 + \left(\frac{\omega}{c}\right)^2}$$

$$= 20 \log \frac{\omega}{c}$$

$\Rightarrow 20 \log\left(\frac{\omega}{c}\right)$

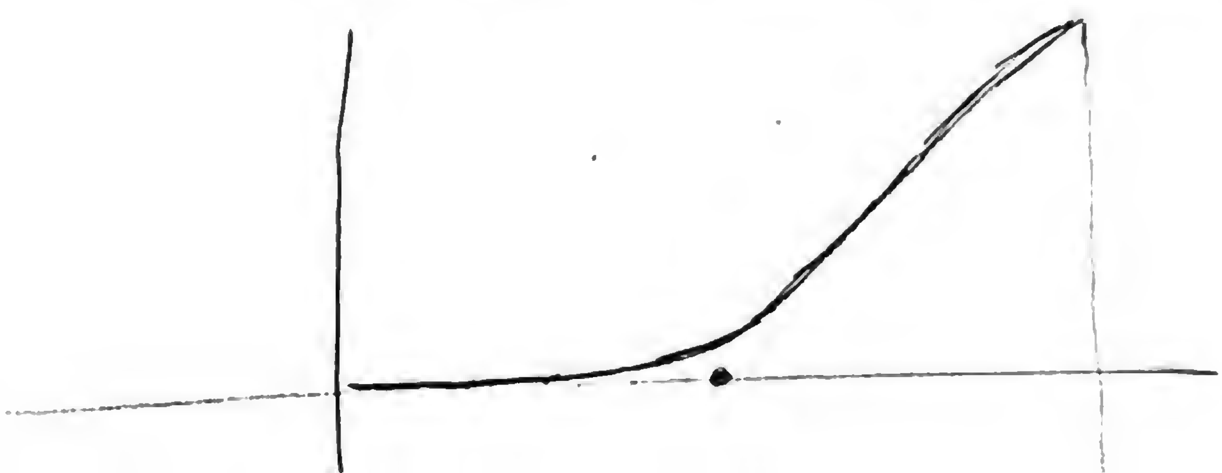
— خط مستقيم يمر بالنقطة $\omega = c$




$C \rightarrow$ corner frequency

— نقطة إنقلاب الميل يعني الميل قبلها بقيمة وبعدها بقيمة تانية.

— لو بتحل exact هتكون كده

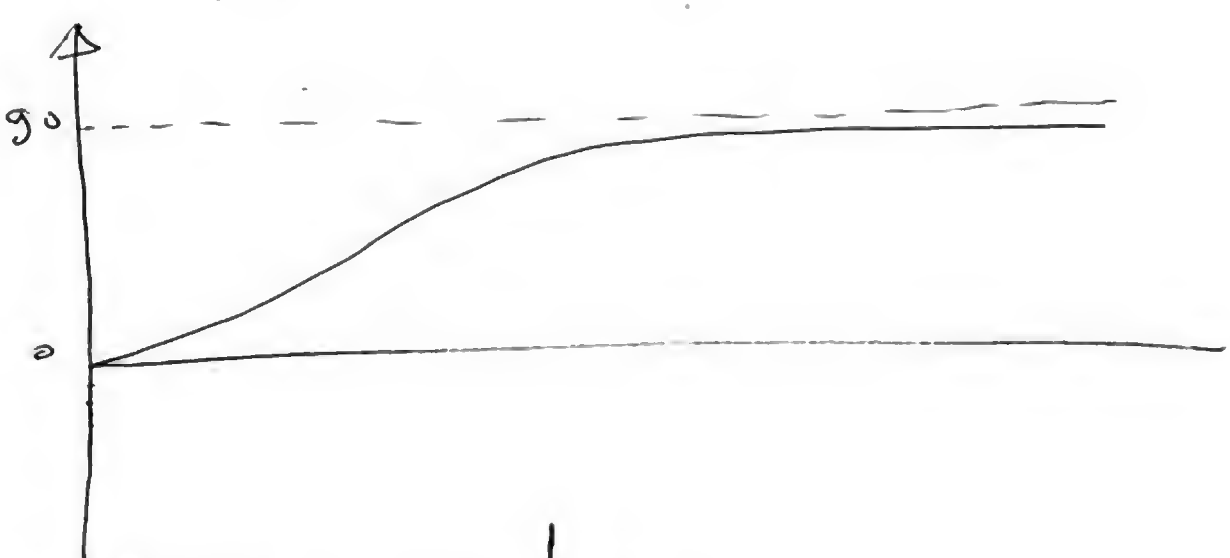


$$\phi(w) = \tan^{-1}\left(\frac{w}{c}\right)$$

<u>Range</u> $\quad 0 \longrightarrow 90$

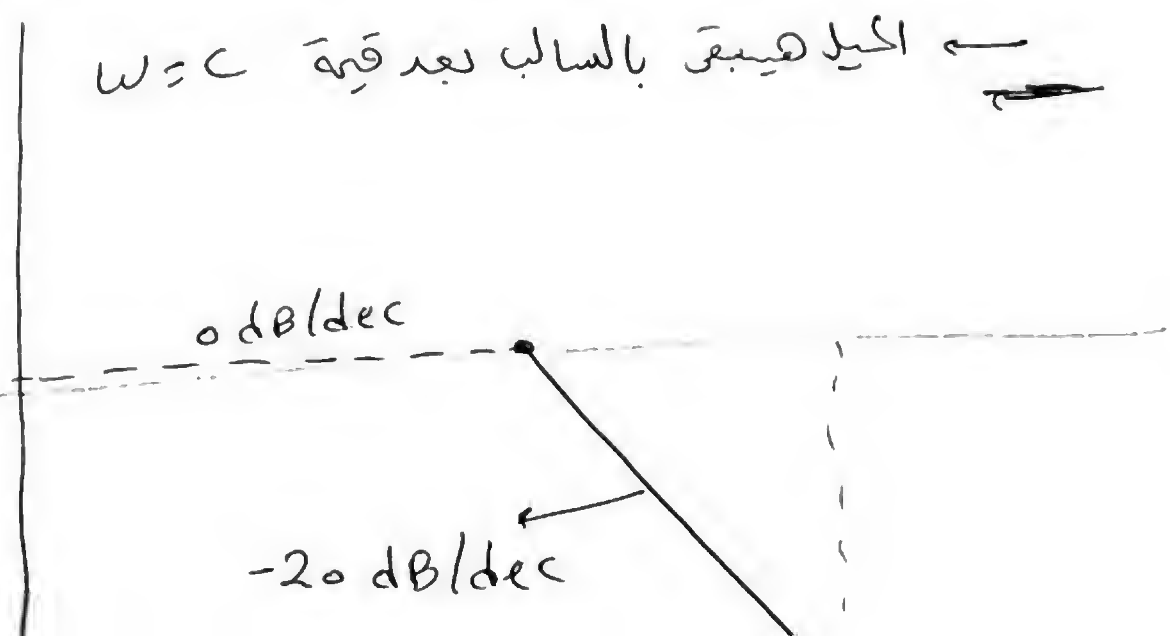
— لو بترسم (exact) هتعمل جدول

| $\omega$ | $0$ | $\infty$ |
|---|---|---|
| $\phi(\omega)$ | $0$ | $90$ |



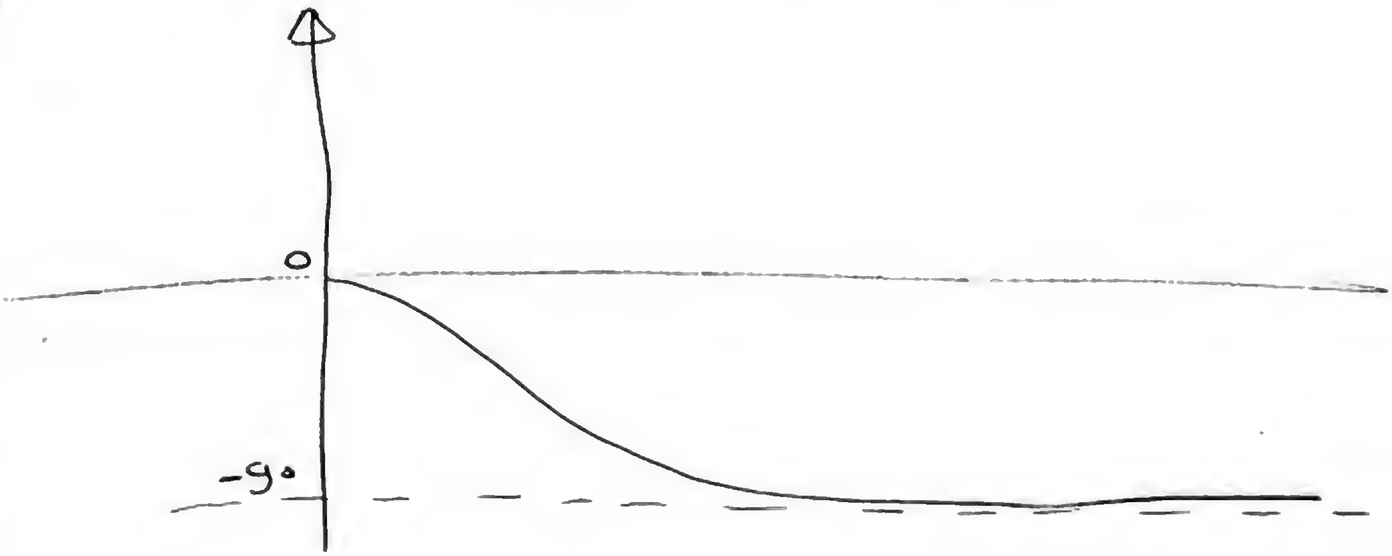

$$* \quad GH(s) = \frac{1}{1 + \frac{s}{c}}$$

— الميل هيبقى بالسالب بعد قيمة $w = c$



$$\phi(\omega) = 0 - \tan^{-1}\left(\frac{\omega}{c}\right)$$

Range of Angle $0 \longrightarrow -90$




$$\left(1 + \frac{s}{c}\right)^{\pm n} = GH(s) \cancel{\pm 20 n \ dB/dec}$$

$c \Rightarrow$ Corner frequency

$0 \ dB/dec = (\omega = c)$ الميل قبل

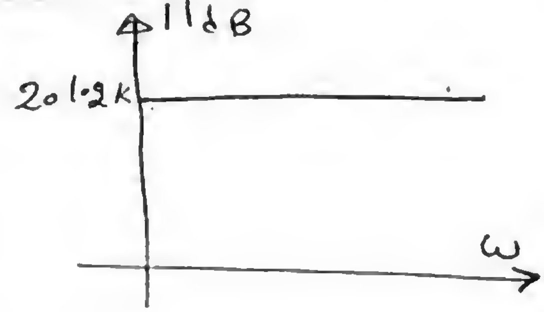
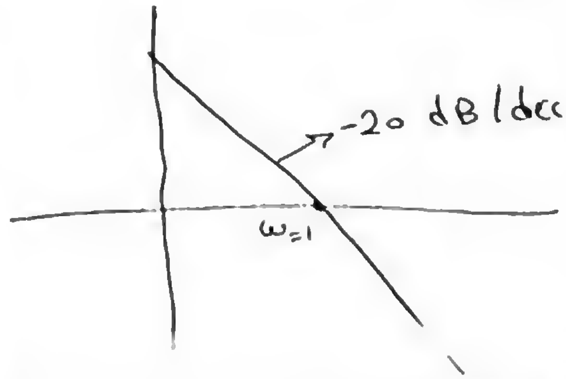
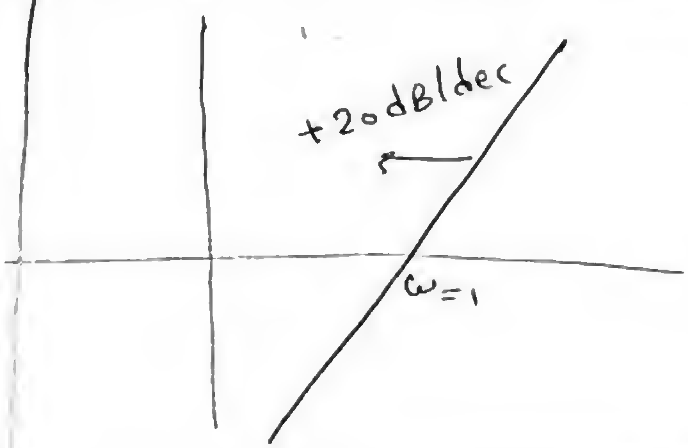
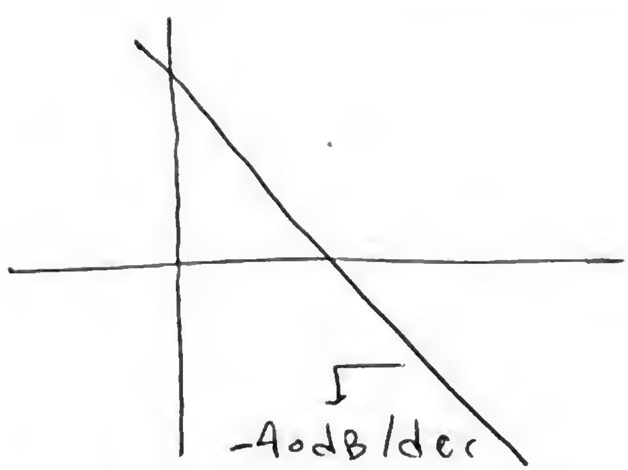
$\pm 20 \ dB/dec = (\omega = c)$ بعد ''

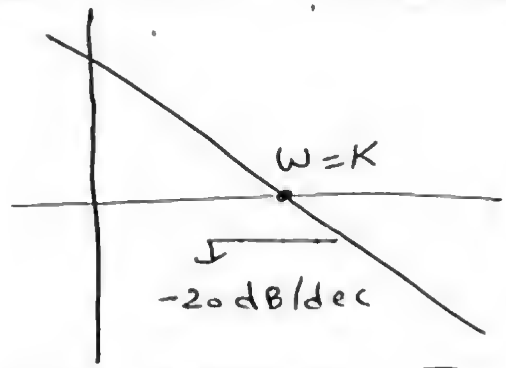
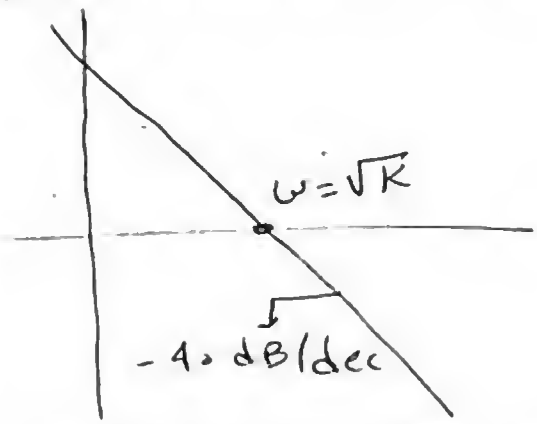
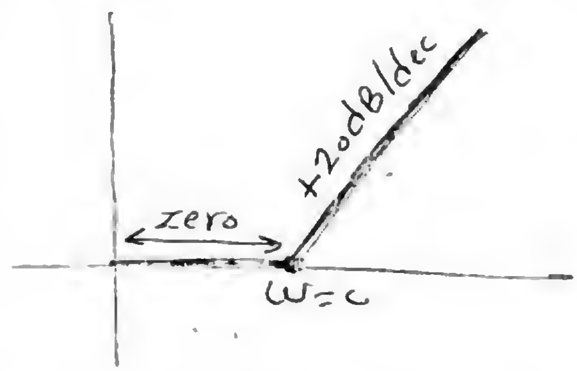
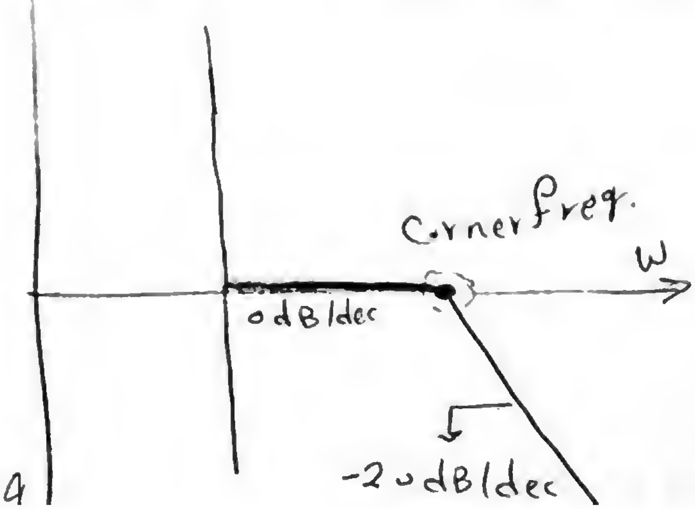
$$\phi(\omega) = \pm n \tan^{-1}\left(\frac{\omega}{c}\right)$$

Range of $\phi(\omega)$ $\rightarrow \omega = 0 \Rightarrow 0$

$\rightarrow \omega = \infty \Rightarrow \pm 90 n$

$|Z|$

| Term | $\phi(\omega)$ | $\lvert \cdot \rvert_{dB}$ |
|---|---|---|
| $K$ | $0$ |   |
| $\frac{1}{s}$ or $\frac{1}{j\omega}$ | $-90$ |   |
| $s$ or $j\omega$ | $+90$ |   |
| $\frac{1}{s^2} \Rightarrow \frac{1}{j\omega \cdot j\omega}$ | $-180$ |   |

| Term | $\phi(\omega)$ | $\| \ \|_{dB}$ |
|---|---|---|
| $\frac{K}{s} \Rightarrow \frac{K}{j\omega}$ | $-90$ |   |
| $\frac{K}{s^2} \Rightarrow \frac{K}{j\omega \cdot j\omega}$ | $-180$ |   |
| $1+\frac{s}{c}$ <br> $\Downarrow$ $1+j\frac{\omega}{c}$ | $\tan^{-1}\left(\frac{\omega}{c}\right)$ |   |
| $\frac{1}{1+\frac{s}{c}}$ <br> $\Downarrow$ <br> $\frac{1}{1+j\frac{\omega}{c}}$ | $-\tan^{-1}\frac{\omega}{c}$ |   |

$\boxed{Ex}$ $$GH(s) = \frac{10}{(1+s)(1+0.1s)}$$

* Draw the bode diagram. & find GM & PM

<u>Sol</u>

$s \rightarrow J\omega$

$$GH(J\omega) = \frac{10}{(1+J\omega)(1+0.1J\omega)}$$

$$= \frac{10}{\left(1+J\frac{\omega}{1}\right)\left(1+J\frac{\omega}{10}\right)}$$

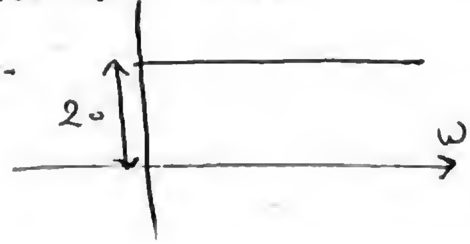
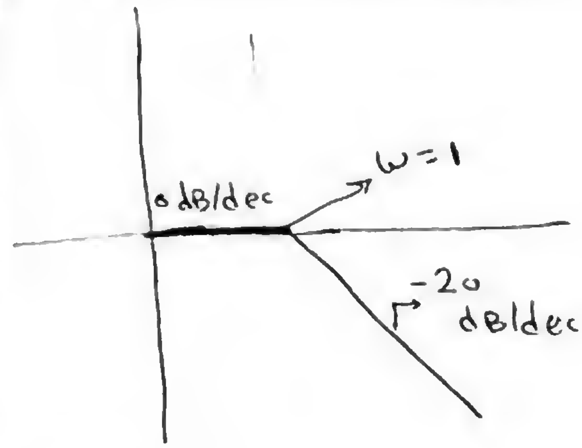
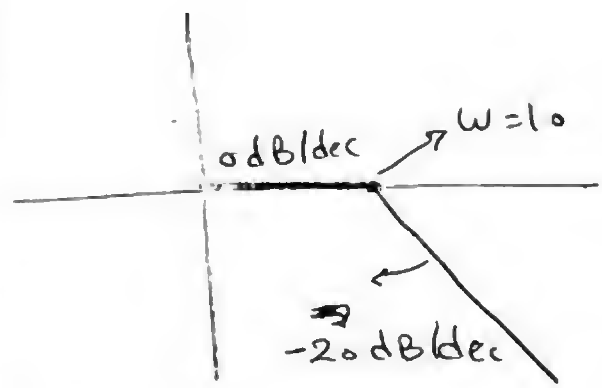
$$\boxed{\phi(\omega) = 0 - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)}$$

البسط

$1+J\frac{\omega}{1}$

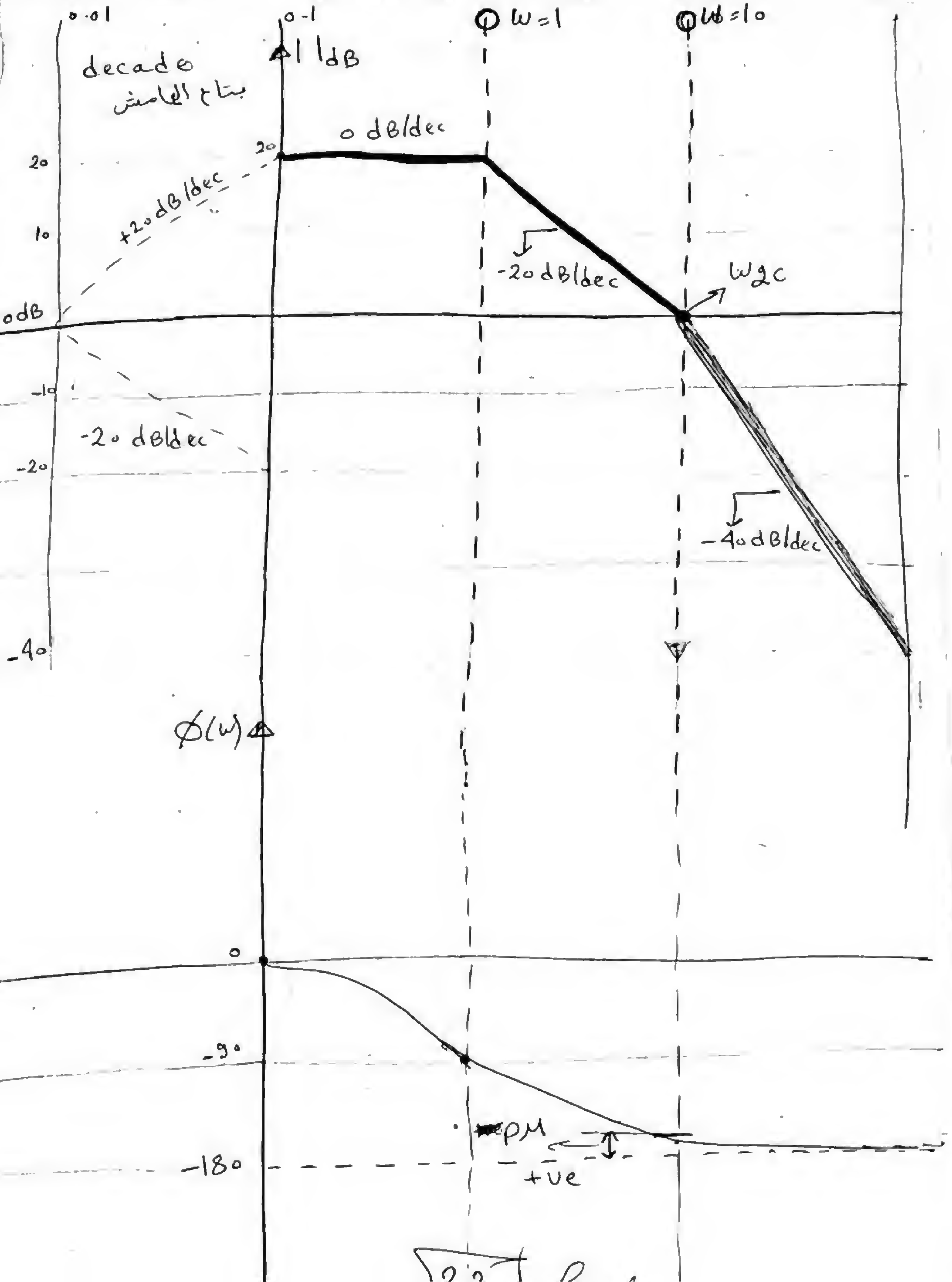
$1+J\frac{\omega}{10}$

~~Fix~~

| Term | $\phi(\omega)$ | $\| \|_{dB}$ |
|---|---|---|
| $10$ | $0$ | $20\log 10 = 20$   |
| $1 + J\frac{\omega}{1}$ | $-\tan^{-1}(\omega)$ |   |
| $1 + J\frac{\omega}{10}$ | $-\tan^{-1}\left(\frac{\omega}{10}\right)$ ⋮ |   |

$$\therefore \phi(\omega) = 0 - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

زي ما قلنا
في الصفحة
السابقة



0.01
0.1
ω=1
ω=10
| |dB
decade
بتاع الهامش
0 dB/dec
20
+20dB/dec
10
-20dB/dec
ωgc
0dB
-10
-20 dB/dec
-20
-40dB/dec
-40
Ø(ω)
0
-90
PM
-180
+ve

$w_{gc}$

| $\omega$ | 0 | 0.1 | 1 | 10 | 100 | $\infty$ |
|---|---|---|---|---|---|---|
| $\phi(\omega)$ | 0 | $-6.3$ | $-50.7$ | $-129.3$ | $-173.7^\circ$ | $-180$ |

$$PM = 180 + \phi(w = W_{gc}) = 180 - 129.3 = --$$

أو تجيبها من الرسم.

$GM = \infty$

عشان لم يحدث تقاطع تحت.

$\longrightarrow$ system stable

$\boxed{23}$ Lec4

الصفحة السابقة تم توضيح الرسم على ورق semilog.